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Collocation Method for the Solution of Boundary Value Problems

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Authors' contributions

This work was carried out in collaboration between two authors. Both authors read and approved the final manuscript.

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Abstract

In mathematics, collocation method is a method for the numerical solution of ordinary differential, partial differential and integral equations. The idea is to choose a finite-dimensional space of candidate solutions (usually polynomial up to a certain degree) and a number of points in the domain (called collocation points), and to select that solution which satisfies the given equation at the collocation points. A numerical method for solving non-linear two-point boundary value problems was implemented which based on collocation method. Two-point Taylor polynomial of order six was used as trial function to obtain the residual function. The method was implemented on some existing problems solved with other numerical methods to show that the method can be equal used to solve the problem, the results obtained were compared to verify the reliability and accuracy of the method and it was observed that collocation method is more effective in each case because the error is minimal compare with the results obtained with the other numerical methods.

Keywords: Non-linear boundary value problem; Two–point Taylor polynomial; collocation method.

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1 Introduction

Most of the scientific problems in Engineering are non-linear, finding the exact solutions of such problems are quiet difficult. Therefore, there have been attempts to develop new techniques for obtaining analytical solutions which reasonably approximate the exact solutions. Researchers have used several methods to tackle this type of problems, these include, Aregbesola, applied Two-point Taylor series for the numerical solution of two-point [1], Kasi and Raju examined Cubic B-Spline Collocation Method for Fourth Order Boundary Value Problems [2], Noor et al. [3] used Variation of Parameter Method, Odejide and Adesina studied the solution of non-linear fifth order boundary value problems [4],Vedat and Hassan, also used the method of Differential Transform [5], Odejide and Aregbesola, worked on Method of Weighted [6], Oderinu and Aregbesola, applied method of Weighted Residual via partition [7], Inayat et al. [8] used New Iterative Method and Shooting Method via Taylor series also applied by Oderinu and Aregbesola [9].

In this paper, collocation method was applied to solving non-linear two-point boundary value problems. The two-point Taylor polynomial of order six was used as trial function because it converges faster than the previous order used by other authors, which on substitution into the differential equation gives the residual function. The residual function was then minimized within the collocation points and the trial function was also forced to satisfy the boundary conditions given to obtain some set of equations.

2 Method of Solution

Definition: Let a and b be two distinct points. If $f(x) \in C^{2m}[a,b]$ *then* $f(x)$ can be approximated by the polynomial $P_{2m-1}(x)$ given as

$$p_{2m-1}(x) = (x-a)^m \sum_{k=0}^{m-1} \frac{B_k (x-b)^k}{k!} + (x-b)^m \sum_{k=0}^{m-1} A_k \frac{(x-a)^k}{k!} = f_m \tag{1}$$

where A_k *and* B_k are constants to be determined which satisfied the given boundary conditions.

and

$$A_k = \frac{d^k}{dx^k}\left[\frac{f(x)}{(x-b)^m}\right]_{x=a}, \qquad B_k = \frac{d^k}{dx^k}\left[\frac{f(x)}{\left(x-a\right)^m}\right]_{x=b} \tag{2}$$

And the remainder is given as

$$f(x) - p_{2m-1}(x) = \frac{f^{2m}(\theta)}{(2m)!}(x-a)^m (x-b)^m \qquad \textit{with} \quad a \le \theta < b. \tag{3}$$

Equation (1) is referred to as two-point Taylor polynomial approximation, representing the evaluated derivatives with constants and with m = 6, the following were obtained.

$$f_6 = (x-a)^6\left[\sum_{k=0}^{5} B_k \frac{(x-b)^k}{k!}\right] + (x-b)^b\left[\sum_{k=0}^{5} A_k \frac{(x-a)^k}{k!}\right]$$

$$f_6 = B_0(x-a)^6 + B_1(x-a)^6(x-b) + B_2(x-a)^6(x-b)^2 + B_3(x-a)^6(x-b)^3 + B_4(x-b)^6(x-a)^4$$
$$+ B_5(x-a)^6(x-b)^5 + A_0(x-b)^6 + A_1(x-b)^6(x-a) + A_2(x-b)^6(x-a)^2 + A_3(x-b)^6(x-a)^3$$
$$+ A_4(x-b)^6(x-a)^4 + A_5(x-b)^6(x-a)^5 \quad (4)$$

Where A_k and B_k are constants to be determined and k = 0, 1. . . 6.

Weighted Residual Method: Suppose we have a differential equation

L(u(x))=f in the domain Ω (5)

$$B_u(u) = \Omega \quad on\ \partial\Omega, \quad (6)$$

where L(u) denotes a general differential operator (Linear or non-linear) involving spatial derivatives of dependent variable u, f is a known function of position. B_u (u) represents the approximate number of boundary conditions and Ω is the domain with the boundary $\partial\Omega$.

Equation (1) is forced to satisfy the boundary conditions, which gives some sets of equations and the residual function is obtained by substituting equation (1) into the original differential equation which was then minimized at the collocation points within the domain. On solving these systems of equations together the constants and the approximate solution were obtained.

3 Numerical Applications

Problem 1 Solve

$$f^{IV}(x) = f^2(x) - x^{10} + 4x^9 - 4x^7 + 8x^6 - 4x^4 + 120x - 48. \qquad [2] \quad (7)$$

Subject to the boundary conditions

$$f(0) = 0; \qquad f(1) = 1 \qquad f'(0) = 0; \qquad f'(1) = 1$$

The exact solution is

$$f(x) = x^5 - 2x^4 + 2x^2.$$

Following the procedure discussed above, the approximate solution was obtained:

$$f_6 = x^6 - 5x^6(x-1) + 15x^6(x-1)^2 - 33x^6(x-1)^3 + 61x^6(x-1)^4 - 100.99x^6(x-1)^5$$
$$+1.99(x-1)^6x^2 + 12(x-1)^6x^3 + 39.99(x-1)^6x^4 + 101(x-1)^6x^5$$

Problem 2: Solve

$$f^{VI}(x) = e^x f^2(x) \qquad [3] \quad (8)$$

Subject to the boundary conditions

$$f(0)=1 \qquad f(1)=e^{-1}, \qquad f'(0)=-1 \qquad f'(1)=-e^{-1}, \quad f''(0)=1 \qquad f''(1)=e^{-1}$$

The exact solution is

$$f(x)=e^{-x}$$

Following the procedures discussed in Problem 1 to obtain,

$$\begin{aligned} f_6 &= 0.3678794412x^6-2.575156088x^6(x-1)+10.11668463x^6(x-1)^2-29.49166854x^6(x-1)^3 \\ &+71.2x^6(x-1)^4-150.7416667x^6(x-1)^5(x-1)^3+71.2x^6(x-1)^4-150.7416667x^6(x-1)^5+(x-1)^6 \\ &+5(x-1)^6x+15.5(x-1)^6x^2+37.833(x-1)^6x^3+79.54166667(x-1)^6x^4+150.7416667(x-1)^6x^5 \end{aligned}$$

Problem 3: Solve

$$32f^V=e^{-x}f^3(x) \qquad [7] \tag{9}$$

Subject to the boundary conditions

$$f(0)=1 \quad f'(0)=\frac{1}{2} \qquad f''(0)=\frac{1}{4}, \qquad f(1)=e^{\frac{1}{2}}, \qquad f'(1)=\frac{1}{2}e^{\frac{1}{2}}$$

The exact solution is

$$f(x)=e^{\frac{x}{2}}$$

Following the procedures discussed in problem1 to obtain,

$$\begin{aligned} f_6 &= 1.64872127x^6-9.067966990x^6(x-1)+29.88307304x^6(x-1)^2-76.21901042x^6(x-1)^3 \\ &+165.7007813x^6(x-1)^4-322.4533854x^6(x-1)+6.5(x-1)^6x+24.125(x-1)^6x^2 \\ &+67.27083333(x-1)^6x^3+156.7526042(x-1)^6x^4+322.4533854(x-1)^6x^5 \end{aligned}$$

Problem 4: Solve

$$f^V=e^{-x}f^2(x) \qquad [4]\&[3] \tag{10}$$

Subject to the boundary conditions

$$f(0)=f'(0)=f''(0)=1 \qquad f(1)=f'(1)=e$$

The exact solution is $f(x) = e^x$.

Following the procedures discussed in Problem 1 to obtain,

$$f_6 = 2.718281828x^6 - 13.59140914x^6(x-1) + 42.13336834x^6(x-1)^2 - 102.8416625x^6(x-1)^3 + 216.2166671x^6(x-1)^4 - 409.7583333x^6(x-1)^5 + 7(x-1)^6 x + 27.5(x-1)^6 x^2 + 80.1666667(x-1)^6 x^3 + 193.5416667(x-1)^6 x^4 + 409.7583333(x-1)^6 x^5$$

Table 1. Comparison of absolute error between the exact and the computed values for problem 1

x	**Exact**	**Collocation method**	**Galerkin with quintic B-spline**	**Cubic B spline collocation**
0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.0	$4.31x10^{-7}$	$4.92x10^{-6}$
0.2	0.0	0.0	$1.81x10^{-6}$	$680x10^{-6}$
0.3	0.0	0.0	$3.58x10^{-6}$	$6.36x10^{-6}$
0.4	0.0	0.0	$4.20x10^{-6}$	$4.35x10^{-6}$
0.5	0.0	0.0	$4.95x10^{-6}$	$1.37x10^{-6}$
0.6	0.0	0.0	$7.09x10^{-6}$	$2.15x10^{-6}$
0.7	0.0	0.0	$6.56x10^{-6}$	$4.65x10^{-6}$
0.8	0.0	0.0	$3.87x10^{-6}$	$6.26x10^{-6}$
0.9	0.0	0.0	$2.80x10^{-6}$	$4.83x10^{-6}$
1.0	0.0	0.0	0.0	0.0

Table 2. Comparison of absolute error between the exact and the computed values for problem 2

x	**Exact**	**Collocation Method**	**DTM(n=17)**
0.0	0.0	0.0	0.0
0.1	0.0	0.0	$1.8x10^{-8}$
0.2	0.0	0.0	$4.7\ x10^{-8}$
0.3	0.0	0.0	$7.9\ x10^{-8}$
0.4	0.0	$1\ x10^{-10}$	$1.5\ x10^{-7}$
0.5	0.0	$1x10^{-10}$	$1.4\ x10^{-7}$
0.6	0.0	0.0	$1.64x10^{-7}$
0.7	0.0	0.0	$2.0x10^{-7}$
0.8	0.0	$1\ x10^{-10}$	$2.3\ x10^{-7}$
0.9	0.0	0.0	$2.4\ x10^{-7}$
1.0	0.0	0.0	$1.6x10^{-7}$

Table 3. Comparison of absolute error between the exact and the computed values for problem 3

x	**Exact**	**Collocation method**	**ADM**	**DTM**
0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.0	$3.32x10^{-5}$	$7.82\ x10^{-6}$
0.2	0.0	$1.0x10^{-9}$	$2.06\ x10^{-4}$	$4.99\ x10^{-5}$
0.3	0.0	0.0	$5.21\ x10^{-4}$	$1.30\ x10^{-4}$
0.4	0.0	0.0	$1.48\ x10^{-3}$	$2.28\ x10^{-4}$
0.5	0.0	0.0	$1.17\ x10^{-3}$	$3.12\ x10^{-4}$
0.6	0.0	0.0	$1.26\ x10^{-3}$	$4.01\ x10^{-4}$
0.7	0.0	0.0	$1.10\ x10^{-3}$	$5.23\ x10^{-4}$
0.8	0.0	0.0	$7.09\ x10^{-4}$	$2.52\ x10^{-4}$
0.9	0.0	$1.0x\ 10^{-9}$	$2.45\ x10^{-4}$	$3.12\ x10^{-4}$
1.0	0.0	0.0	$1.40x10^{-8}$	$1.20\ x10^{-7}$

Table 4. Comparison of absolute error between the exact and the computed values for problem 4

x	Exact	Collocation method	VIM	HPM	VOP
0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.0	$1.0x10^{-9}$	$1.0 x10^{-9}$	$1.3 x10^{-12}$
0.2	0.0	0.0	$2. 0x10^{-9}$	$2.0 x10^{-9}$	$1.0 x10^{-11}$
0.3	0.0	0.0	$1. 0x10^{-8}$	$1.0 x10^{-8}$	$3.2 x10^{-11}$
0.4	0.0	$1.0x10^{-9}$	$2. 0x10^{-8}$	$2.0 x10^{-8}$	$7.0 x10^{-11}$
0.5	0.0	0.0	$3.1x10^{-8}$	$3.1 x10^{-8}$	$1.2 x10^{-10}$
0.6	0.0	0.0	$3.7 x10^{-8}$	$3.7 x10^{-8}$	$1.9 x10^{-8}$
0.7	0.0	$1.0x10^{-9}$	$4.1 x10^{-8}$	$4.1 x10^{-8}$	$2.8 x10^{-10}$
0.8	0.0	$1.0x10^{-9}$	$3.1 x10^{-8}$	$3.1 x10^{-8}$	$3.7 x10^{-10}$
0.9	0.0	0.0	$1.4 x10^{-8}$	$1.4 x10^{-8}$	$4.7 x10^{-10}$
1.0	0.0	0.0	0.0	0.0	0.0

Error =/exact-approximate/

The comparison of the absolute error between the exact values and the computed values of each method in the literature were tabulated in above tables and it was shown that collocation method was the same with the exact solution at some points and the error at other points was better in comparison with other methods.

4 Conclusion

In this paper, two-point Taylor polynomial of order six was used as trial function in using collocation method to solve non-linear two-point boundary value problems. The computed results were compared with other methods referenced and the exact solution. From the Problems considered the method was more accurate than other methods such as Adomian Decomposition Method, Differential Transform Method, Variational Iterative Method, Variation of Parameter Method, Homotopy Perturbation Method, Galerkin with Quintic B-Spline and Cubic B-Spline Collocation. This is evident from the error of the computational results obtained. The method has the sole advantage of presenting the solution in polynomial forms.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Aregbesola YAS. Two-point Taylor series for the numerical solution of two-point Boundary value problems. Analele Stiintifice Ale Universitatii Ali Cuza Lasi XLII, S.L.A Mathematica. 1996;71-77.

[2] Kasi KNS, Raju Showri Y. Cubic B-spline collocation method for fourth order boundary value problems. International Journal of Non-linear Science. 2012;14(3):336-344.

[3] Noor MA, Mohyud-Din ST, Waheed A. Variation of parameter method for solving fifth order boundary value problems. Journal of Applied Mathematics and Information Science. 2008;2(2):135-141.

[4] Odejide SA, Adesina OA. A note on the solution of non-linear fifth order boundary value problems. International Journal of Non-linear Science. 2012;13(4):419-421.

[5] Vedat SE, Hassan IHA. Solutions of different types of linear and non-linear higher order boundary value problems by differential transform method. European Journal of Pure and Applied Mathematics. 2009;2(3):426-447.

[6] Odejide SA, Aregbesola YAS. Application of method of weighted residual to problems with infinite domain. ROM, Journal of Physics. 2011;56(1-2):14-24. Bucharest.

[7] Oderinu RA, Aregbesola YAS. Two-point Taylor series for two-point higher order boundary value problems using weighted residual method. International Journal of Applied Mathematics. 2012;25(2): 207-218.

[8] Inayat U, Hamid K, Rahim M. Numerical solutions of higher order non-linear boundary value problems by New Iterative Method Applied Mathematical Sciences. 2013;7(49): 2429-2439.

[9] Oderinu RA, Aregbesola YAS. Shooting method via Taylor series for solving two-point boundary value problems on an infinite interval. Gen. Mathematics Notes. 2014;24(1):74-83.

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