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A Proposed Numerical Integration Method Using Polynomial Interpolation

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Authors' contributions

This work was carried out in collaboration between all authors. Author KAD conceptualized the methodology of the study and also took part in the analysis. Authors ENNN and CAL worked on the literature review and also took part in the data analysis. All authors read and approved the final manuscript.

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Abstract

The main aim of this paper is to propose a numerical integration method using polynomial interpolation that provides improved estimates as compared to the Newton-Cotes methods of integration. The method is an extension of trapezoidal rule where the Lagrange interpolation is employed when fitting polynomials after segmentation. We proved that the proposed numerical integration method using polynomial interpolation provided an improved formula for numerical integration. The proposed method using polynomial interpolation gave better estimates as compared to some Newton-Cotes methods of integration.

Keywords: Numerical integration; polynomial interpolation; Newton-Cotes methods; relative error.

1 Introduction

Integration is a way of assessing the area under a function plotted on a graph. The application of numerical integration has several application in various fields such as statistics, actuarial science, engineering finance, etc. [1,2,3,4,5].

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The approximation of general functions by simple classes of functions has many applications as well as theoretical implications (page 183 in [6]). The approximation of numerical values that cannot be integrated analytically is referred to as numerical integration by Gordon (page 1 in [7]). Various methods of numerical integration such as Gauss Quadrature, Newton- Cotes, Monte Carlo integration and Romberg are mostly employed to compute those functions that are not easily integrated. Numerical integration has been used in several areas of science such as biostatistics to estimate several distribution functions and quantiles and in economics to estimate the Lorenz curve when computing the Gini coefficient of income [8]. Using Bayesian methods, numerical integration is recently employed in estimating likelihoods and posterior distributions [9]. In the era of modern computers, approximation via interpolation has emerged as a general paradigm for computing elementary functions as part of typical system software on current computers [10]. Scott [6] stated that the earliest applications of interpolation was simply to link scattered data to provide some sense of what a continuum representation might look like. Taylor theorem in calculus provides a polynomial approximation to sufficiently smooth function [11]:

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k \tag{1}$$

Taylor theorem is very powerful and requires knowing the values of high-order derivatives of f to construct $P_n(x)$. Interpolation is a more distributed approximation that does not require derivatives, only the values of f (page 203 in [6]).

Interpolation addresses the approximation of a function which is known through its nodal values to make it easier to integrate or differentiate. Given m+1 pairs (x_i, y_i), interpolation consists of identifying a function $P(x)$ such that $P(x_i) = f(x_i)\,\text{for}\, i = 0,1,...,m$ with $f(x_i)$ being some values given, then we can conclude that $P(x)$ interpolates $P(x_i) = f(x_i)$ at the nodes x_i (page 327 in [12]). There are several types of interpolation. They are Polynomial interpolation, Trigonometric interpolation and Spline or Piecewise polynomial interpolation. Polynomial interpolation is derived if $P(x)$ is an algebraic polynomial, trigonometric interpolation if $P(x)$ is a trigonometric polynomial or piecewise polynomial interpolation (or spline interpolation) if $P(x)$ is only locally a polynomial. The numbers $P(x_i) = f(x_i)$ may represent the values attained at the nodes x_i by a function $f(x)$ that is known in closed form, as well as experimental data [12]. The primary goal of approximation is to provide a compact representation of the available data, whose number is often quite large. For polynomial interpolation, we consider $n+1$ pairs (x_i, y_i) in other to find a polynomial $P(x)$ such that

$$P(x_i) = a_m x_i^m + ... + a_1 x_1 + a_0 = f(x_i), i = 0,1,2,...,n \tag{2}$$

There are several forms of Polynomial interpolation. They are linear, Quadratic and Lagrange polynomial interpolation.

The Linear polynomial interpolation is the simplest form of interpolation in the form of a straight line, connecting two points. Considering two known data points $(x_0, y_0)\,\text{and}\,(x_1, y_1)$, there exist a unique straight line passing through these points. The formula for a straight line can be written as $P_1(x) = a_0 + a_1 x$. Using the linear polynomial interpolation to write $P_1(x)$, we have

$$
\begin{aligned}
P_1(x) &= \frac{x-x_1}{x_0-x_1} f(x_o) + \frac{x-x_0}{x_1-x_0} f(x_1) \\
&= \frac{(x_1-x)f(x_0)+(x-x_0)f(x_1)}{x_1-x_0} \\
&= f(x_0) + \frac{x-x_0}{x_1-x_0}\left[f(x_1)-f(x_0)\right] \\
P_1(x) &= f(x_0) + \left(\frac{f(x_1)-f(x_0)}{x_1-x_0}\right)(x-x_0)
\end{aligned}
\tag{3}
$$

The Quadratic polynomial interpolation satisfies $P_2(x_i) = y_i, i = 0,1,2$ for given data points $(x_0, y_0), (x_1, y_1), (x_2, y_2)$.

Here there exist a unique quadratic curve passing through these points. The formula for a quadratic curve can be written as $P_2(x) = a_0 + a_1 x + a_2 x^2$. Using the quadratic polynomial interpolation to write $P_2(x)$, we have

$$P_2(x) = f(x_0)L_0(x) + f(x_1)L_1(x) + f(x_2)L_2(x) \tag{4}$$

With

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}, L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}, L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \tag{5}$$

The formula in (2) is called Lagrange's form of the interpolation polynomial.

The Lagrange interpolant is defined by

$$P_n(x) = \sum_{i=0}^{n} f(x_i)\,\mathrm{L}_i(x) \tag{6}$$

where $n \text{ in } P_n(x)$ stands for the order polynomial that approximates the function $y = f(x)$ given at $n+1$ data points as $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ and $\mathrm{L}_i(x) = \prod_{k=0,\, k\neq i}^{n} \frac{x-x_k}{x_i-x_k}, i = 0,1,2,\ldots,n$

where each $L_i(x)$ is the Lagrange Polynomial.

The Newton-Cotes formula is a frequently used interpolator function that employs Lagrange interpolating when fitting polynomials. Letting $x_0 = a, x_n = b \, and \, h = \frac{(b-a)}{n}$, the Newton-Cotes methods are derived by integrating $p(x) \text{ over} [a,b]$ and choosing $x_i = a + \frac{(b-a)i}{n}$. That is

$$\int_a^b p(x)dx = \sum_{i=0}^{n}\left[\int_{x_0}^{x_1} f(x_i)\prod_{k=0,k\neq i}^{n}\frac{x-x_k}{x_i-x_k}dx, i=0,1,2,...,n\right] \quad (7)$$

When $n=1,$ we have a simple trapezoidal rule of [13]

$$A_2 = \frac{h}{2}[f(x_0)+f(x_1)] \quad (8)$$

When $n=2,$ it gives the Simpson $1/3$ rule of [13]

$$A_3 = \frac{h}{3}[f(x_0)+4f(x_1)+f(x_2)] \quad (9)$$

When $n=3,$ it gives the Simpson $3/8$ rule of [13]

$$A_4 = \frac{3h}{8}[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)] \quad (10)$$

When $n=4$, it gives the Boole's rule of [13]

$$A_5 = \frac{2h}{45}[7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4)] \quad (11)$$

When $n=6$, it gives the Weddle's rule of [13]

$$A_6 = \frac{3h}{10}[f(x_0)+5f(x_1)+f(x_2)+6f(x_3)+f(x_4)+5f(x_5)+f(x_6)] \quad (12)$$

Chasnov [14] and Yang, Cao, Chung and Morris ([15]) have explained into details the Romberg rule, Richardson's extrapolation, Gauss quadrature, Euler method and so forth.

The main objective of this paper is to propose a numerical integration method using polynomial interpolation that provides improved estimates as compared to the Newton-Cotes methods of integration.

2 Materials and Methods

2.1 Proposed numerical integration formula using polynomial interpolation

Suppose the interval $[a,b]$ is subdivided into $n(n\in z_+)$ equal interval length $h=\frac{b-a}{n}$. Define x_i by $x_i = a+ih, i=0,1,2,...,n.$ Then $x_0=a$ and $x_n=b$. Let $f_i = f(x_i), i=0,1,2,...,n$ be the ordinate at $x_i(i=0,1,2,...,n)$ of the function f. Suppose also that the interval $[x_i,x_{i+1}], i=0,1,2,...,n-1$ is divided into k equispaced points $x_i+\frac{t}{k}h, t=1,2,...,k$: then the corresponding ordinates of f are given by

$f_{i+\frac{t}{k}} = f(x_i + \frac{t}{k}h), t = 1, 2, ..., k; i = 0, 1, 2, ..., n-1.$ Clearly when $t = k, x_i + \frac{k}{k}h = x_{i+1}$ and $f_{i+\frac{k}{k}} = f_{i+1.}$

Approximating the area under the curve $y = f(x)$ from $x = a$ to $x = b$, we employ Lagrange interpolating when fitting polynomials [13]. Letting $x_0 = a, x_n = b \, and \, h = \frac{(b-a)}{n}$, we have

$$f(x) \approx \sum_{i=0}^{n} f(x_i) p_i(x) \tag{13}$$

Integrating $f(x)$ over $[a,b]$ and choosing $x_i = a + \frac{(b-a)i}{n}$ we have the proposed numerical integration method

$$A_i^* = \int_a^b f(x) \approx \sum_{i=0}^{n} w_i f(x_i) \tag{14}$$

$$A_i^* = \sum_{t=1}^{k} \int_a^b \left[f(x_{i+\frac{t-1}{k}}) \frac{x - x_{i+\frac{t}{k}}}{x_{i+\frac{t-1}{k}} - x_{i+\frac{t}{k}}} + f(x_{i+\frac{t}{k}}) \frac{x - x_{i+\frac{t-1}{k}}}{x_{i+\frac{t}{k}} - x_{i+\frac{t-1}{k}}} \right] dx, i = 0, 1, 2, ..., n; t = 1, 2, ..., k \tag{15}$$

where the weight w_i is determined by

$$w_i = \int_a^b p_i(x) dx = \int_a^b \frac{x - x_{i+\frac{t}{2}}}{x_{\frac{t-1}{2}} - x_{i+\frac{t}{2}}} + \frac{x - x_{i+\frac{t-1}{2}}}{x_{\frac{t}{2}} - x_{i+\frac{t-1}{2}}} dx, i = 0, 1, 2, ..., n; t = 1, 2, ..., k \tag{16}$$

The area under the curve for the i^{th} strip is estimated as [13]

$$A_i^* = \frac{h}{2k} \left[f(x_i) + 2 \sum_{t=1}^{k-1} f_{i+\frac{t}{k}} + f(x_{i+1}) \right], i = 0, 1, ..., n-1; k = 1, 2, ... \tag{17}$$

When $k = 1$, we have [13]

$$A_i^* = \frac{h}{2} [f_i + f_{i+1}] \tag{17a}$$

When $k = 2$, we have [13]

$$A_i^* = \frac{h}{4}\left[f_i + 2f_{i+\frac{1}{2}} + f_{i+1} \right] \tag{17b}$$

When $k = 3,$ it becomes [13]

$$A_i^* = \frac{h}{6}\left[f_i + 2\left\{ f_{i+\frac{1}{3}} + f_{i+\frac{2}{3}} \right\} + f_{i+1} \right] \tag{17c}$$

2.1.1 Proof

Here we provide the proof for $k = 2$. We can similarly derive the proof of $k \geq 3$. We have left out the proof for the case of $k = 1$ because it is the same as the Trapezium Rule [13]. Now, when using interpolation, when $k = 2$ an estimate of the area of the i^{th} strip under the curve is given by

$$A_i^* = \sum_{t=1}^{2} \int_{x_{i+\frac{t-1}{2}}}^{x_{i+\frac{t}{2}}} \left[f(x_{i+\frac{t-1}{2}}) \frac{x - x_{i+\frac{t}{2}}}{x_{i+\frac{t-1}{2}} - x_{i+\frac{t}{2}}} + f(x_{i+\frac{t}{2}}) \frac{x - x_{i+\frac{t-1}{2}}}{x_{i+\frac{t}{2}} - x_{i+\frac{t-1}{2}}} \right] dx \tag{18}$$

$$A_i^* = \int_{x_i}^{x_{i+\frac{1}{2}}} \left[f(x_i) \frac{x - x_{i+\frac{1}{2}}}{x_i - x_{i+\frac{1}{2}}} + f(x_{i+\frac{1}{2}}) \frac{x - x_i}{x_{i+\frac{1}{2}} - x_i} \right] dx + \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \left[f(x_{i+\frac{1}{2}}) \frac{x - x_{i+1}}{x_{i+\frac{1}{2}} - x_{i+1}} + f(x_{i+1}) \frac{x - x_{i+\frac{1}{2}}}{x_{i+1} - x_{i+\frac{1}{2}}} \right] dx$$

$$A_i^* = f(x_i) \int_{x_i}^{x_{i+\frac{1}{2}}} \frac{x - x_{i+\frac{1}{2}}}{x_i - x_{i+\frac{1}{2}}} dx + f(x_{i+\frac{1}{2}}) \int_{x_i}^{x_{i+\frac{1}{2}}} \frac{x - x_i}{x_{i+\frac{1}{2}} - x_i} dx +$$

$$f(x_{i+\frac{1}{2}}) \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{x - x_{i+1}}{x_{i+\frac{1}{2}} - x_{i+1}} dx + f(x_{i+1}) \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{x - x_{i+\frac{1}{2}}}{x_{i+1} - x_{i+\frac{1}{2}}} dx$$

Given $h = x_{i+\frac{1}{2}} - x_i = x_{i+1} - x_{i+\frac{1}{2}}$ to be the interval length, then the new interval length will be $\frac{h}{2}$ for $k = 2$. Substituting $u = -x + x_{i+\frac{1}{2}}$ into the first integral gives

$$f(x_i)\int_{x_i}^{x_{i+\frac{1}{2}}}\frac{x-x_{i+\frac{1}{2}}}{x_i-x_{i+\frac{1}{2}}}dx = f(x_i)\int_{h/2}^{0}\frac{-u}{-(h/2)}(-du)$$

$$= f(x_i)\int_{0}^{h/2}\frac{u}{(h/2)}(du) = f(x_i)\frac{h}{4}$$

For the second integral after substituting $u = x - x_i$ is

$$f(x_{i+\frac{1}{2}})\int_{x_i}^{x_{i+\frac{1}{2}}}\frac{x-x_i}{x_{i+\frac{1}{2}}-x_i}dx = f(x_{i+\frac{1}{2}})\int_{0}^{h/2}\frac{u}{(h/2)}du = f(x_{i+\frac{1}{2}})\frac{h}{4}$$

For the third integral after substituting $u = -x + x_{i+1}$ is

$$f(x_{i+\frac{1}{2}})\int_{x_{i+\frac{1}{2}}}^{x_{i+1}}\frac{x-x_{i+1}}{x_{i+\frac{1}{2}}-x_{i+1}}dx = f(x_{i+\frac{1}{2}})\int_{h/2}^{0}\frac{-u}{-(h/2)}(-du)$$

$$= f(x_{i+\frac{1}{2}})\int_{0}^{h/2}\frac{u}{(h/2)}(du) = f(x_{i+\frac{1}{2}})\frac{h}{4}$$

For the fourth integral after substituting $u = x - x_{i+\frac{1}{2}}$ is

$$f(x_{i+1})\int_{x_{i+\frac{1}{2}}}^{x_{i+1}}\frac{x-x_{i+\frac{1}{2}}}{x_{i+1}-x_{i+\frac{1}{2}}}dx = f(x_{i+1})\int_{0}^{h/2}\frac{u}{(h/2)}du = f(x_{i+1})\frac{h}{4}$$

Therefore $\int_{x_i}^{x_{i+1}} f(x)dx \approx f(x_i)\frac{h}{4} + f(x_{i+\frac{1}{2}})\frac{h}{4} + f(x_{i+\frac{1}{2}})\frac{h}{4} + f(x_{i+1})\frac{h}{4}$

$$= \frac{h}{4}\left[f(x_i) + 2f(x_{i+\frac{1}{2}}) + f(x_{i+1}) \right]$$

Example

The exact method and the other numerical integration methods stated will be employed as an illustration to estimate the integral below;

$$\int_{0}^{2} 0.05e^{x^3}dx$$

Solution

Method 1: Exact (Numerical Integration)

$$\int_0^2 0.05e^{x^3}dx = \int_0^2 0.05\sum_{n=0}^{\infty}\frac{x^{3n}}{n!}dx$$
$$= \sum_{n=0}^{\infty} 0.05\int_0^2 \frac{x^{3n}}{n!}dx$$
$$= 0.05\sum_{n=0}^{\infty}\frac{2^{3n+1}}{(3n+1)n!}$$
$$= 0.05\left[2+\frac{2^4}{4}+\frac{2^7}{7(2!)}+\frac{2^{10}}{10(3!)}+\ldots+\frac{2^{37}}{37(12!)}+\ldots\right]$$
= 13.8426444

Now, estimating the integral of the function using several numerical integration methods such as the Trapezium rule, Simpson $\frac{1}{3}$ rule, Simpson $\frac{3}{8}$ rule, Boole's rule and Weddle's rule, we select n=12 divisions of the interval [0,2] . Given $h = \frac{b-a}{n} = \frac{2-0}{12} = 0.16667$, the corresponding ordinates of $x_i = a + ih$ are $f_i = 0.05e^{x_i^3}\ (i = 0,1,2,\ldots,12)$ as displayed in Table 1 in the appendix section.

Method 2: Trapezium rule

The estimate of the integral by the trapezium rule is given by

$$= \frac{0.16667}{2}\left[1+2(0.0502+0.0519+\ldots+5.1238+23.7197)+149.0479\right]$$
$$= \frac{1}{12}\left[0.05+2(63.07114)+149.0479\right]$$
$$= \frac{212.169}{12} = 17.68075$$

Method 3: Simpson's $\frac{1}{3}$ rule

Using Simpson's $\frac{1}{3}$ rule, the estimate of the integral is given by

$$= \frac{0.16667}{3}\left[0.05+4(0.0502+0.0567+0.0892+0.2447+1.4612+23.7197)+\right.$$
$$\left.2(0.0519+0.0672+0.1359+0.5351+5.1238)+149.0479\right]$$
$$= \frac{1}{18}\left[1+4(25.6217)+2(5.9139)+149.0479\right]$$
$$= \frac{263.4123}{18} = 14.63402$$

Method 4: Simpson's $3/8$ rule

By the Simpson's $3/8$ rule, the estimate of the integral is

$$= \frac{3(0.16667)}{8}\left[0.05 + 3(0.0502 + 0.0519 + 0.0672 + 0.0892 + 0.2447 + 0.5351 + 5.1238 + 23.7197) + 2(0.0567 + 0.1359 + 1.4612) + 149.0479\right]$$

$$= \frac{1}{16}\left[1 + 3(29.8818) + 2(1.6538) + 149.0479\right]$$

$$= \frac{121.0254}{16} = 15.1282 .$$

Method 5: Boole's rule

Using Boole's rule, the estimate is

$$= \frac{2(0.16667)}{45}\left[7(0.05 + 149.0479) + 32(0.0502 + 0.0567 + ... + 1.4612 + 23.7197) + 12(0.0519 + 0.1359 + 5.1238) + 14(0.0672 + 0.5351)\right]$$

$$= \frac{1}{135}\left[7(149.0979) + 32(25.6217) + 12(5.3116) + 14(0.6023)\right]$$

$$= \frac{1935.75}{135} = 14.33889$$

Method 6: Weddle's rule

Using Weddle's rule, the estimate is

$$= \frac{3(0.16667)}{10}\left[0.05 + 5(0.0502 + 0.2447) + (0.0519 + 0.5351) + 6(0.0567 + 1.4612) + (0.0672 + 5.1238) + 5(0.0892 + 23.7197) + 2(0.1359) + 149.0479\right]$$

$$= \frac{1}{20}\left[0.05 + 5(24.1038) + 6(1.5179) + 2(2.7183) + 5.7780 + 149.0479\right]$$

$$= \frac{284.7739}{20} = 14.2387$$

Method 7: Proposed Numerical Integration Method

Using the proposed numerical integration method, the values of x_i and $x_{i+\frac{1}{2}}$ and their corresponding ordinates $f_i = 0.05e^{x_i^3}$ and $f_{i+\frac{1}{2}} = 0.05e^{x_{i+\frac{1}{2}}^3}$ are displayed in Table 1 in the appendix. For the proposed numerical integration method, the values are based on $k = 2$ since $k = 1$ is the same as the Trapezium rule. An estimate of the integral using the proposed method when $k = 2$ is given as follows

$$= \frac{(0.1667)}{4}\left[0.05 + 2(0.0500 + 0.0502 + 0.0508 + ... + 57.1315) + 149.0479\right]$$

$$= \frac{1}{24}\left[0.05 + 2(103.7333) + 149.0479\right]$$

$$= \frac{356.5644}{24} = 14.8569$$

Generally, when $k = 8, 9, ..., 20,$ the values for the Proposed Integration method using interpolation for evaluating $\int_0^2 0.05e^{x^3}\,dx$ taking 12 divisions are given in Table 1.

Table 1. Estimates of the area using both classical and proposed numerical integration method with 12 divisions

Integration method	Area	Absolute error	Relative error (%)	M
Exact (Numerical integration)	13.84264	-		
Trapezium rule	17.68075	3.83811	27.72672	1
Simpson's 1/3	14.63402	0.79138	5.716973	2
Simpson 3/8	15.12818	1.28554	9.286812	1
Boole's rule	14.33889	0.49625	3.584938	2
Weddle's rule	14.23869	0.39605	2.861087	2
Proposed Method when k=8	13.90725	0.06461	0.466746	3
Proposed Method when k=9	13.89371	0.05107	0.368933	3
Proposed Method when k=10	13.88401	0.04137	0.298859	3
Proposed Method when k=11	13.87684	0.03420	0.247063	3
Proposed Method when k=12	13.87138	0.02874	0.207619	3
Proposed Method when k=13	13.86713	0.02449	0.176917	3
Proposed Method when k=14	13.86376	0.02112	0.152572	3
Proposed Method when k=15	13.86104	0.01840	0.132923	3
Proposed Method when k=16	13.85881	0.01617	0.116813	3
Proposed Method when k=17	13.85697	0.01433	0.103521	3
Proposed Method when k=18	13.85542	0.01278	0.092323	3
Proposed Method when k=19	13.85411	0.01147	0.082860	3
Proposed Method when k=20	13.85299	0.01035	0.074769	4

**m = the number of significant digits at least correct*
Source: Author's research

3 Results and Discussion

From Table 1, the Proposed Numerical Integration method using interpolation is compared to the various numerical integration formulas using the relative errors. When $k = 1$, the area under the curve is 17.68075 which gives the same result as the area under the curve using the Trapezium rule. Also when $k = 2$ and above, the estimates of the area under the curve are better than the Trapezium rule. Furthermore, when k = 3 and above, the estimates of the area under the curve are better than the Simpson $1/3$ and $3/8$ rule. When $k = 8$ and above, the estimates of the area under the curve are better than Boole and Weddle's rule. The proposed method using interpolation with smaller divisions of the interval gives a better estimate with lesser errors as compared to the Trapezium rule, Simpson $1/3$ and $3/8$ rule, Boole's and Weddle's rule.

4 Conclusion

This study proposes a numerical integration method using polynomial interpolations. The proposed numerical integration method using polynomial interpolation was used to estimate the area under a curve and has proven to be better than the classical methods of numerical integration such as Trapezium rule, Simpson 1/3 and 3/8 rules, Boole rule and Weddle rule.

Competing Interests

Authors have declared that no competing interests exist.

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