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The Impact of Solar Flares and Cosmic Rays on Atmospheric Decay

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Author's contribution

This whole work carried out by the author MMK.

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ABSTRACT

Aims: The objective of this paper is to introduce a Relativistic Stochastic Process that calculates the number of cosmic ray and high energy solar flare particles that enter the atmosphere.
Study Design: The Relativistic Stochastic Process is tested against the Arley model.
Place and Duration of Study: Department of research and development, at Economics Traffic Clinic in Paris, between January 2013 and March 2013.
Methodology: The Relativistic Stochastic Process is a derivative of the Arley's probabilistic model. Although Arley's simplified model gives satisfactory results, it does not adequately address the phenomenon of cosmic ray and high energy solar flare particle showers.
Results: The simulation with the Relativistic Stochastic Process gives better results compared to the Arley model.
Conclusion: Relativistic Stochastic Process is more realistic and thus accurate and reliable.

Keywords: Cosmic rays; solar flares; atmosphere; probabilistic differential equations; random walk; velocity; energy-momentum tensor; electromagnetic force.

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1. INTRODUCTION

The objective of this paper is to introduce a Relativistic Stochastic Process (R.S.P.) that calculates the number of cosmic ray and high energy solar flare particles that enter the atmosphere. (R.S.P.) is a derivative of the Arley's [1,2], probabilistic model. The Arley model is based on the probabilistic system of recursive differential equations where the solution to the PDE (Probabilistic Differential Equations) is given by the Poisson distribution. Arley made two major assumptions that simplified his model and gave explicit solutions. He assumed that the probabilities of absorption of each particle increase linearly in time, and the probabilities of multiplication remain constant. Though Arley's simplified model gives satisfactory results, it does not adequately address the phenomenon of cosmic ray and high energy solar flare particle showers. There is a good chance that the solution (the probability of (n) particles entering the atmosphere) is either higher or lower than what is calculated by the simplified Arley PDE model. The accuracy of estimation depends on many factors such as the particle's initial energy-momentum level, and particle velocity. The natural extension to the model is a more complicated stochastic process, [3] where in addition to time, other adjustments are made. In order to deal with the complexities involved in modeling, the simplified assumptions of the Arley model are modified. In the (R.S.P.) model it is assumed that the probability of particle absorption is a random walk and the probability of particle energy multiplication is a function of the velocity and the angle from which particles enter the atmosphere.

Edward P. Ney in 1959 [4,5] put forward the idea of the role of cosmic rays either directly or via solar turbulence and wind induce climate change. Cosmic rays and solar flares in the form of large energy release and the stream of X-ray and Extreme Ultra Violet (EUV) rays affect the Earth's atmosphere. The impact is notably observed in cloud formation and thus cooling of the climate, annual cloud cover variations, weakening of monsoon rainfalls and vice versa, formation of relatively highly charged aerosols and cloud droplets at cloud boundaries, production of ultra-small aerosol particles orders of magnitude smaller than cloud condensation nuclei, (CCN), total and spectral solar irradiance, and finally sudden flares produce large amounts of X-rays and EUV energy. The complications due to the impact of solar flares and cosmic rays on the atmosphere have lead to the recent development of cosmic ray ionization models following the works of Ney, notably the works of Svensmark and colleagues,[6,7,8] and many other researchers [9,10,11,12].

Many techniques are used to detect and estimate the flux of cosmic ray particles that enter the earth's upper atmosphere and eventually hit the surface of the earth. For example, cosmic rays can be detected using particle detectors aboard satellites or high altitude balloons. The detection is mainly done using a technique developed by Fleischer, Price, and Walker, [13] which consists of exposing a sheet of polycarbonate resin thermoplastic to space or high altitudes and count the number of hits. So far, there exist very basic stochastic models that attempt to predict the number of cosmic ray particles that enter the earth's atmosphere. The stochastic processes used come in the form of a system of partial differential equations; in particular, Markov processes where all future probability relations are entirely determined by the present state probabilities. Typically, these probabilities indicate the occurrence of continuous events where the changes from the current state occur during any time interval, however small, but given the probability laws, in small time intervals the changes are small.

There is a need for a reliable mathematical model to estimate the expected number of particles that enter the atmosphere. Systems of stochastic partial differential probabilities do not seem to adequately describe the phenomenon of cosmic ray showers and solar flares impact on the environment. The estimated expected number of particles entering the earth's atmosphere would either regularly increase or regularly decrease. This is in contrast to the observations. According to many published experiments, [14,15,16] the number of electrons first rapidly increases due to high energy levels and then rapidly decreases when approaching the earth's atmosphere as the particles lose their initial momentum and thus energy. To describe this dynamic more complicated models of stochastic processes are needed. Complication comes from the fact that the state of the dynamic system cannot be described by a simple integer. The Arley model is an attempt to obtain more realistic results. He postulates that: 1. the probability of particle absorption is linearly monotonically increasing in time, 2. the probability of particle multiplication remains constant in time. Though the Arley model produces more or less acceptable results, there is still room for improvement. The improvement comes in the form of: the Relativistic Stochastic Process or (R.S.P.). To implement the R.S.P. model new assumptions are required. The new assumptions are: 1. the probability of particle absorption is a random walk, 2. the probability of particle multiplication is a function of the velocity and the angle from which particles enter the atmosphere. This process consists of replacing the intensities of particle absorption and multiplication with their relativistic equivalences. The relativistic energy of particle absorption ($\tilde{\gamma}$) is defined as a function of relativistic energy-momentum tensor ($T_{(R,g)}$) and the electromagnetic force (B):

$$\tilde{\gamma} = T_{(R,g)} - B \tag{1}$$

(R) is the Riemann tensor, and (g) is the gravitational force.

The relativistic energy-momentum tensor ($T_{(a,b)}$) is generally expressed by the Einstein's field equation as:

$$\kappa_0 T_{(a,b)} = R_{(a,b)} - \frac{1}{2} R \times g_{(a,b)} + \Lambda \times g_{(a,b)} \tag{2}$$

(κ_0) is the gravitational constant, (λ) is the cosmological constant, ($R_{(a,b)}$) is the Riemann tensor, ($R = R_a^a$) is the scalar curvature. It is posited that the particles ejected from the center of the solar eruption have maximum energy momentum and their trajectory towards earth will be a 1-form Riemann tensor (R) and thus have the greatest chance of absorption and multiplication. In general, cosmic rays particles retain their maximum energy momentum in space if not impacted by gravitational zones, and thus would follow a 1-form Riemann tensor trajectory. The 1-form Riemann tensor ($R^a_{(b,c)}$) is expressed by:

$$R^a_{(b,c)} = R^a_{(b,c)} \omega^c = \Gamma^a_{(b,c)} \omega^c \tag{3}$$

The relativistic energy of particle multiplication ($\tilde{\eta}$) is the relativistic energy-momentum tensor ($T_{(R,g)}$), per unit of relativistic time (τ_R) as:

$$\tilde{\eta} = \left(\frac{T_{(R,g)}}{\tau_R} \right) \tag{4}$$

The relativistic time (τ_R) is expressed as:

$$\tau_R = \frac{t - \frac{v}{c^2} \times R}{\sqrt{t - \frac{v^2}{c^2}}} \tag{5}$$

In Equation (5), (v) represents particle velocity, (c) is the speed of light, and (R) is the Riemann tensor. The relativistic intensity of particle absorption ($\tilde{\zeta}$) is defined as the relativistic energy of particle absorption ($\tilde{\gamma}$) multiplied by the relativistic time (τ_R) multiplied by the number of particles (n).

$$\tilde{\zeta} = n \times \tilde{\gamma} \times \tau_R \tag{6}$$

The relativistic intensity of particle multiplication ($\tilde{\vartheta}$) is the relativistic energy of particle multiplication ($\tilde{\eta}$) multiplied by the number of particles (n).

$$\tilde{\vartheta} = \tilde{\eta} \times n \tag{7}$$

The Relativistic Stochastic Process (R.S.P.) takes up the structure of the Arley model and modifies it using the relativistic variables introduced in this section.

2. ARLEY MODEL

One particular stochastic Markov process is the Poisson process used by Arley in determining the number of cosmic rays. A simple assumption is made, and that is to keep the energy of particle multiplication (η) constant meaning ($\eta_i = \eta$),(i) represents each state of particle multiplication and absorption during the particle trajectory into the earth's atmosphere. Similarly the energy of absorption ($\gamma_i = \gamma$), where (γ) is the energy of particle absorption. Assuming that the probability of particle absorption is an increasing function of time and that the probability of particle multiplication should stay constant during any time interval, this leads to the intensity of (n) particle absorption being equal to ($n \times \gamma \times t$), and the intensity of (n) particle multiplication being equal to ($n \times \eta$). Given this premise, a system of stochastic differential equations can be constructed that takes the following form, [17,18].

$$\begin{aligned} P_0^{'}(t) &= \gamma \times t P_1(t) \\ P_n^{'}(t) &= -n(\eta + \gamma \times t)P_n(t) + (n-1) \times \eta P_{n-1}(t) + (n+1) \times \gamma \times t P_{n+1}(t) \end{aligned} \tag{8}$$

The natural initial conditions are ($P_0(0) = 1$), and ($P_n(0) = 0$) for ($1 \le n$). The expected number of particles M(t) entering the atmosphere during any interval (t) is given by:

$$M(t) = e^{\left(\eta \times t - \left(\frac{\gamma}{2}\right) \times t^2\right)} \tag{9}$$

The maximum expected number of particles is given by ($t = \frac{\eta}{\gamma}$). Following the method of generating functions, Arley produces an explicit solution for the system of equations (8) in the form of:

$$P_0(t) = 1 - e^{\left(-\eta \times t + \frac{\gamma}{2} \times t^2\right)}$$
$$P_n(t) = e^{\left(-\eta \times t + \frac{\gamma}{2} \times t^2\right) \times A^{(n-1)} \times \left(A + e^{\frac{(\gamma \times t - \eta)^2}{2 \times \gamma}}\right)^{-(n+1)}} \quad (10)$$

Where

$$A = \eta \times \int_0^t e^{\frac{(\gamma \times s - \eta)^2}{2 \times \gamma}} ds \quad (11)$$

Though the system of equation (8) and the solution agree with a certain number of experimental results, they do not adequately describe the phenomenon of cosmic rays showers. The discrepancy between the observational data and the outcome of the Arley model is due to the main assumptions of the model, notably the intensities of particle multiplication and absorption. The intensity of particle multiplication increases linearly as a function of time, and the intensity of absorption increases with the number of cosmic rays and solar flares particles. The Arley model uses as input the average number of particles entering the atmosphere at any time (t) which further explains the shortcomings of the model. The results of the Arley model simulation runs when compared to actual observational data are either overestimated or underestimated.

3. RELATIVISTIC STOCHASTIC PROCESS (R.S.P.)

The system of Equations (8) does not adequately describe the phenomenon of cosmic rays. Given the overall pattern, the expected number of particles would either constantly increase or constantly decrease, whereas in reality, the number of electrons first rises rapidly, then it decreases, taken into account that with increasing depth of atmospheric permeation the particles lose energy so are absorbed rapidly. Another shortcoming of Eq. (8) is pointed out by Arley himself which is that the actual fluctuations in the number of cosmic ray entry into the atmosphere are much larger than what is calculated by Eq. (8). Arley believed that the actual probabilities of particle multiplication after entering the atmosphere and consequently particle absorption depend on the particle's energy level, on the magnitude of the solar wind, on the initial velocities, the trajectories of the cosmic ray particles before and after entering the atmosphere, the velocities, and the trajectories of the solar wind. Arley, concluded that to adequately replicate the actual situation, more complicated stochastic processes are required.

Arley, did not propose a complicated stochastic process model, he suggested a simple modification to (8). He assumed that the probability of particle absorption in the atmosphere increases linearly with time, while the probability of particle multiplication stays constant. The new stochastic probability model that will be introduced is built over the Arley model, is more complicated, since the structure relies on two major factors: the velocities and the trajectories of cosmic rays and the velocities and the trajectories of the solar wind. This structure by necessity depends entirely on relativistic concepts of curvature and time. The details are given in this section. The Arley model provides a reasonable and simplistic model of cosmic rays and solar flares particles entry into the atmosphere. Many aspects of the model can be kept such as the initial boundary conditions ($P_0(0)=1$) and ($P_n(0)=0$) for ($1 \leq n$) the differential probability system of equations (8) can be retained in the form proposed by Arley, because it is a correct basis model for theoretical calculations. The assumption that the energy of particle multiplication and absorption is constant cannot be kept due to the relativistic transformations of these variables. The relativistic stochastic system is the modification of the Arley model where the time interval (t) is replaced by the relativistic time (τ_R) and the intensities of particle absorption (γ) and multiplication (η) are replaced by their relativistic equivalences ($\tilde{\gamma}$) and ($\tilde{\eta}$). In this section, it is assumed that (R), the Riemann curvature remains a 1-form for all particles approaching and entering the earth. Other major difference with the Arley model is the replacement of the simple probability $\left(P_n\left(\tau_R\right)\right)$ with the Bayesian probability $\left(P_n\left(\tau_R \middle| R\right)\right)$.

$$\begin{aligned} &P_0^{'}\left(\tau_R \middle| R\right)=\left(\tilde{\gamma}\times\tau_R\right)\times P_1\left(\tau_R \middle| R\right)\\ &P_n^{'}\left(\tau_R \middle| R\right)=-n\times\left(\tilde{\eta}+\left(\tilde{\gamma}\times\tau_R\right)\right)P_n\left(\tau_R \middle| R\right)+\left(\left(n-1\right)\times\tilde{\eta}\right)P_{n-1}\left(\tau_R \middle| R\right)+\left(\left(n+1\right)\times\tilde{\gamma}\right)\times\tau_R P_{n+1}\left(\tau_R \middle| R\right) \end{aligned} \tag{12}$$

$\left(P_n\left(\tau_R \middle| R\right)\right)$ is the Bayesian (conditional) probability of observing (n) events during the relativistic time interval (τ_R), given that the Riemann tensor (R) is a 1-form.

$$P_n\left(\tau_R \middle| R\right)=\frac{P_n\left(\tau_R \cap R\right)}{P_n\left(R\right)} \tag{13}$$

The relativistic intensity of absorption ($\tilde{\zeta}$) is given as:

$$\tilde{\zeta}=\left(n\times\tilde{\gamma}\times\tau_R\right) \tag{14}$$

Where the relativistic energy of absorption is expressed as:

$$\tilde{\gamma}=T_{R,g}-B \tag{15}$$

(B) is the electromagnetic force and $\left(T_{R,g}\right)$ can be calculated given the following Equation:

$$\kappa_0 T_{a,b}=R_{a,b}-\frac{1}{2}\times R\times g_{a,b}+\lambda\times g_{a,b} \tag{16}$$

The relativistic time (τ_R) is expressed as:

$$\tau_R = \frac{t - \frac{v}{c^2} \times R}{\sqrt{t - \frac{v^2}{c^2}}} \tag{17}$$

The relativistic intensity of particle multiplication ($\tilde{\vartheta}$) is expressed in a similar way as the Arley model:

$$\tilde{\vartheta} = \tilde{\eta} \times n \tag{18}$$

The relativistic intensity of multiplication ($\tilde{\eta}$) is a function of the energy-momentum tensor $\left(T_{R,g}\right)$ and relativistic time $\left(\tau_R\right)$.

$$\tilde{\eta} = \frac{T_{(R,g)}}{\left(\tau_R\right)} \tag{19}$$

Thus

$$\tilde{\vartheta} = \left(\frac{T_{(R,g)}}{\tau_R}\right) \times n \tag{20}$$

In order to find an exact solution to the stochastic system of Equations (12), the energies ($\tilde{\eta}$),($\tilde{\gamma}$) and the intensities ($\tilde{\vartheta}$),($\tilde{\zeta}$) of particle multiplication and absorption are estimated by setting the relativistic time (τ_R) equal to a historical time ($t_{(\text{hist})}$) when the relativistic energy-momentum tensor $\left(T_{R,g}\right)$ and electromagnetic force (B) are observed to be at maximum. This allows for fixed values of ($\tilde{\eta}$),($\tilde{\gamma}$) and ($\tilde{\vartheta}$),($\tilde{\zeta}$) which are based on historical observations. The first Equation of (12) can be rewritten given the relativistic intensity of absorption ($\tilde{\zeta}$), Equation (14) as:

$$P_0^{'}\left(\tau_R \middle| R\right) = \left(\tilde{\zeta}\right) \times P_1\left(\tau_R \middle| R\right) \tag{21}$$

Equation (21) can be rewritten expanding the Bayesian probability $\left(P_n\left(\tau_R \middle| R\right)\right)$ as:

$$P_0^{'}\left(\tau_R \middle| R\right) = \left(\tilde{\zeta}\right) \times \frac{P_1\left(\tau_R \cap R\right)}{P(R)} \tag{22}$$

Given the probability space (Ω) and a probability measure (P) on a Borel field (B) of subsets of (Ω), there exists a pair (Ω, B), a probability measure (P) on (B) and a Borel subfield $((T) \subset (\mathrm{B}))$, such that any function $(Q(\omega, R), \omega \in (\Omega), R \in (\mathrm{B}))$ has the following properties: For any fixed (R),(Q) is a T-measurable function of (ω), and for any fixed (ω) a probability measure on $((\mathrm{B}))$ [19,20,21,22,23]. Thus for any $(T \subset (T))$ and $(R \subset (\mathrm{B}))$, there exists

$$P(T \cap R) = \int_0^{\tau_R} Q(\omega, R) dP(\omega) \tag{23}$$

It is assumed that the expected number of particles entering earth is formulated as Equation (9) in the section on the Arley model, except that the energies of particle multiplication (η) and absorption (γ) are replaced by their relativistic equivalences. Given this, the function $(Q(\omega, R))$, with (R), a constant Riemann 1-form is given as:

$$Q(\omega, R) = e^{\left(-\tilde{\eta} \times \omega + \frac{\tilde{\gamma}}{2} \times \omega^2\right)} \tag{24}$$

Equation (23) can be written as:

$$P(\tau_R \cap R) = \int_0^{\tau_R} e^{\left(-\tilde{\eta} \times \omega + \frac{\tilde{\gamma}}{2} \times \omega^2\right)} dP(\omega) \tag{25}$$

Given Equation (25), the solution to the first part of the system of Equations (12) can be calculated as:

$$P_0\left(\tau_R | R\right) = 1 - P(R)^{-1} \times e^{\left(-\tilde{\eta} + \frac{\tilde{\gamma}}{2} \times \tau_R{}^2\right)} \tag{26}$$

For the case of (n) particle entry, the solution to the Relativistic Stochastic Process is based on the method of generating functions and is similar to the solution given by Arley. Notice that in R.S.P. the variable time (τ_R) is independent of the relativistic energy of multiplication and absorption.

$$P_n\left(\tau_R | R\right) = P(R)^{-1} \times e^{\left(-\tilde{\eta} \times \tau_R + \frac{\tilde{\gamma}}{2} \times \tau_R{}^2\right) \times A^{(n-1)} \times \left(A + e^{\frac{\left(\tilde{\gamma} \times \tau_R - \tilde{\eta}\right)^2}{2 \times \tilde{\gamma}}}\right)^{-(n+1)}} \tag{27}$$

Where

$$A = \tilde{\eta} \times \int_0^{\tau_R} e^{\frac{(\tilde{\gamma} \times s - \tilde{\eta})^2}{2 \times \tilde{\gamma}}} ds \tag{28}$$

4. RSP Vs ARLEY

In this section the Arley model is compared with the Relativistic Stochastic Process (R.S.P.) for the case of zero and (n) particles entries. For the purpose of comparison, in the case of the R.S.P. model the following parameters are chosen: ($\kappa_0 = 1$),($\tau_R = 1$) (hr), ($\lambda = 1$), n=592, $(v = 80\times140 \quad \text{km/s})$, the speed of light $(c = 300\times105 \quad \text{km/s})$, the Riemann tensor $(R_{abcd} = 0)$, the scalar curvature $(R = 1)$, and the electromagnetic force (B=0.3 amp). The results of the application of the R.S.P. model gives the following probabilities for (0) and (n) particles entries as: The probability of zero particle entry $(P_0(\tau_R|R) = 0.7224263)$, and $(P_n(\tau_R|R) = 0.5)$. The results of the R.S.P. test check and confirm experimental observations. For ($0 \le n \le N$),(N large), the relativistic probability $(P_0(\tau_R|R))$, the probability of particle absorption in $(0, \tau_R)$ is at maximum, and the relativistic probability of (n) events $(P_n(\tau_R|R))$ in $(\tau_R, \tau_R + \Delta_{\tau_R})$ decreases steadily to zero as the relativistic time (τ_R) increases. To test the Arley model, the energy of multiplication and absorption are taken to be $(\eta = 1)$, and $(\gamma = 1)$. The probability of zero particle entry is found to be $(P_0(t) = 0.3934693)$, and the probability of (n) particles entry $(P_n(t) = 1)$. The Arley model predicts the probability of absorption in (0,t) to be low, while the probability of occurrence of (n) events in $(t, t + \Delta t)$ is one. This would imply an over-estimation when $(n \le N)$ and an under-estimation when (n=0). Fig. 1 illustrates the probabilities $(P_n(t))$ and $(P_n(\tau_R|R))$ of (n) cosmic rays particles entry for both the Arley and the (R.S.P.) models for $(0 \le n \le 10000)$.

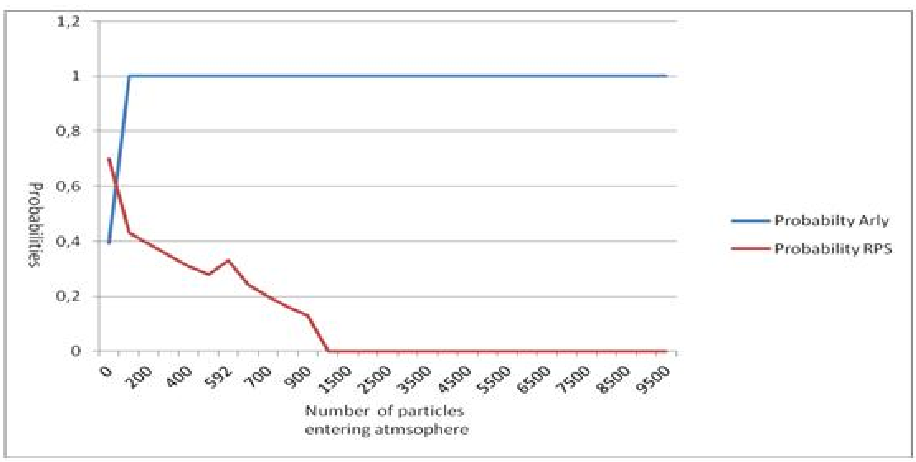

Fig. 1. Comparison of the R.P.S. the Arley simulation results for $(0 \le n \le 10000)$

5. CONCLUSION

This paper proposes a Relativistic Stochastic Process (R.S.P) model which is a modification of the Arley model. The modifications consist of : (1) Changing time from a simple variable (t), with an affine trajectory to a non-linear relativistic variable (τ_R). The relativistic energy of absorption ($\tilde{\gamma}$) and multiplication ($\tilde{\eta}$) replace their equivalences (γ) and (η). ($\tilde{\gamma}$) and ($\tilde{\eta}$) are assumed to be functions of the energy-momentum ($T_{(R,g)}$) tensor, the Riemann (R) tensors, and the relativistic time (τ_R). Though the modifications introduce some complications, the new R.S.P. is an attempt to render the stochastic models used more realistic and thus the probability that at any time interval (t), (n) particles enter the atmosphere are more credible. The conclusion derived from this exercise is that the energy-momentum of particles, the Riemann tensor, and the relativistic time parameters play an important role in the calculation of the probabilities.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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